Computation and the Levi-Civita field

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Abstract:
Elements of the Levi-Civita field are functions from the additive abelian group of rational numbers to the real numbers field, with left-finite support. This lecture will begin with a review of the algebraic and order structures of the Levi-Civita field. Following that we will introduce the Tulliotools library which implements the Levi-Civita field in the C++ programming language. We show that this software can replicate the results of (Shamseddine, 2015) by finding high order derivatives of certain functions faster than commercial software. In addition, we show how the ability to quickly and accurately compute high order derivatives can be combined with Darboux's formula to perform numerical integration. We compare the performance of this new approach to numerical integration with more conventional approaches as well as commercial software and show promising results with regards to both speed and accuracy. We conclude with a discussion of future possibilities in this vein of research.